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Tentative Level-Inflation in Line-Drawings Reconstruction

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Abstract

Optimisation Approaches are used to reconstruct solid models from perspective (non-orthographic) Line Drawings. However, they fail to obtain the global optimum required to succeed in the reconstruction, unless complex tuning in the optimisation algorithms is done.

Tentative models are being introduced to prevent optimisation from falling into local minima. In this paper, a heuristic approach to obtain one of such tentative models is proposed. The heuristic is based on perceptual rules, and gives good results for a large class of polyhedral objects.

Keywords: Document and Text Processing. *Graphics recognition and interpretation.* Artificial Intelligence. *Perceptual reasoning*

1. Introduction

Since the pioneer work by Roberts [1], Line Drawing Reconstruction is seen as a sort of artificial perception problem. In the words of Hoffmann [2], "we do create what we see". Hence, perceiving is constructing mental objects or scenes from signals or cues found in an image.

Visual Perception is assumed to act in a tentative and iterative way. "Organization forces", guide the brain to interpret the figure, and successive organizational attempts are carried out, until the forces are minimized. When contradictory interpretations appear, the brain looks for the best-organized form, and discards those forces been incompatible with the prevailing form. Consequently, an iterative process where some initial solution is refined according with some perceived characteristics, is a good strategy to get what the sole application of geometry rules cannot obtain: a 3D psychologically plausible model. This is why Reconstruction can be described in terms of a mathematical optimisation problem.

Cues are three-dimensional information contained in two-dimensional figures. Mathematical formulation of cues, so-called "regularities", is the way to make explicit human perceptions. Perception psychologists borrowed the term from classic geometry, thus the first appearances of the term regularity associated with graphical representations are due to the Gestalt Psychologists, who named regularities to "those relations that cannot be an accident". Latter on, when some Scientific Visualization areas, related with semantic perception, began their own research, the term regularities became synonymous of "template to describe images in a compact or convenient form". Finally, in the geometrical reconstruction field, regularities are interpreted as "those properties of the image that must correspond to some properties the searched model also has".

Consequently, it is supposed that the properties describing "how the model must be" can be discovered by inspecting the departure image. Then, those regularities can be formulated in a mathematical language [3], and the Objective Function can be easily defined as a weighted sum of regularities [4].

In general, only one solution is "good" in terms of psychological plausibility. That is, human observers seem to have no doubts to choose the appropriate solution. However, mathematical formulation of cues has only been faced by a reduced group of reconstruction community researchers [5]. Hence, at the present, regularities led to a poorly defined objective function, which causes global minimum to be very difficult to obtain. Even Global optimisation algorithms do fail.

Of course, an appropriate taxonomy of regularities would be a general approach to solve the problem. Nevertheless, a simple heuristic solution can be more efficient in some particular cases, because, departing from a good tentative model, even a simple optimisation algorithm with poorly defined Objective Function should find the global optimum.

2. Related approaches

Automatic line drawings reconstruction has been a research area in the Computer Vision field for almost 40 years. The works by Sugihara [6], Nagendra and Gujar [7] and Wang and Grinstein [8], are good references to introduce the earlier advances.

Three approaches can be considered as direct antecedents to optimisation: Edge Labelling, Linear programming, and Perceptual.

Labelling approaches [9-14] are not true reconstruction processes; but its methodology can be helpful to detect "candidate" vertices; edges and faces in any given graph. Consequently, they can be useful to detect "perceptual relations" in the graph.

Linear programming [15-19] fails because its formulation is not tolerant to faults. Nevertheless, bring us the important idea that reconstructing is something like "looking in the graph for a set of independent geometrical conditions (expressed in algebraic notation) the model must fit".

Finally, Perceptual approaches [20] show an effective way to introduce perceptions and invariants (or "regularities") in algebraic formulations. However, the approach lacks a "natural" way to weight and balance different perception rules, and forces all perceptions to appear in the final model.

Optimisation approach has evolved from the ideas by Marill[3], Braid and Wang [21], Leclerc and Fischler[4], and Lipson and Sphitalni [5,22,23].

As was said elsewhere [24], we did make some improvements, mainly in the tuning of optimisation process to get a more robust and automatic process. We did also explore a global minimum optimisation algorithm [25]. We introduced Simulated Annealing algorithms because they claim to be able to find global minimum. Nevertheless, our experience was discouraging because global algorithms fail (as much a local do), when departing from a bad initial model, and when poor or ill-defined regularities are used to generate an Objective Function unnecessarily complex and prone to local minima.

Consequently, the success of optimisation approaches may depend on a correct tuning of a *global optimisation strategy*, or a very careful choice and weighting of all the regularities involved in the process. Nevertheless, we do believe than a simple optimisation from a tentative model is more efficient in some particular cases.

The best antecedent we could find comes from Lipson and Shpitalni [5], who proposed a "preliminary reconstruction" based on the angular distribution graph analysis for obtaining prevailing angles and main directions. When three angles are obtained, a perceptual rule is applied to obtaining a tentative model.

To sum up, tentative inflation approaches allow defining an initial model by applying some simple rules. These approaches proved efficient for a wide class of polyhedral models, and are quite easy to implement and run fast even for complex models [26].

3. Optimisation approach

Optimisation approach is a two-step strategy: an inflation process is used to transform 2D images into 3D models, and one minimisation process is used to determine the best 3D model.

3.1. Inflation process

Only wireframe representations of polyhedral objects ("graph-like" drawings) are considered during tentative level-inflation. In other words, drawings must be made of line-segments (interpreted as projections of edges) and junctions (projections of vertices). The term *junction* (or *2D vertex)* refers to a point were one line-segment ends, or two or more line-segments meet in the drawing. *Line-segments*, or simply *lines*, are the elements connecting two junctions in the drawing, while *edges* are the elements connecting two vertices in the model.

As the general point of view assumption must apply, two univocal correspondences exist between vertices in the model and junctions in the drawing, and between edges in the model and line segments in the drawing. Each junction in the graph represents one and only one vertex in the model, and each line in the drawing represents one and only one edge in the model. Consequently, edges do connect those vertices whose corresponding junctions are connected by its corresponding line-segment.

The inflation is supposed to be the inverse of an orthogonal projection (figure 1). Thus, to establish a simple relation between junctions in the drawing and vertices in the model, a Cartesian coordinate system is defined, where the XY coordinate plane is made coincident with the drawing plane. Then inflation is easily defined to ensure (X, Y) coordinates of every junction in the drawing to be equal to (X, Y) coordinates of the corresponding vertex in the model. Thus, each model obtained by inflation is simply characterized by a set of coordinates $z = (z_1, z_2,...,z_n)$. Where the number of vertices n determines the *order* of the problem.

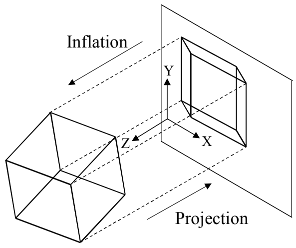


Figure 1: *Inflation of a wireframe from a line drawing.*

3.2. Optimisation

Projection is a univocal transformation, but its “pseudo inverse”, inflation, is not univocal. An infinite number of geometrically valid models can be projected in the same figure.

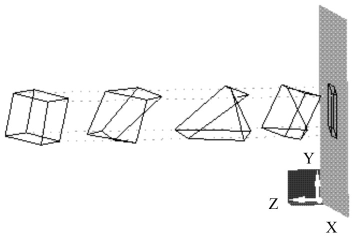


Figure 2: *Some models of the orthographic extension of a line drawing.*

Marill [3] called *orthographic extension* to the full set of three-dimensional objects whose orthogonal projection equals the given drawing. Some of the models belonging to the orthographic extension of a 2D line drawing are shown in figure 2.

It follows that some criteria to choose one particular model among the infinite ones contained in the Extension must be defined. As far as regularities can be formulated in a mathematical language, one Objective Function (F) can be easily defined as a weighted sum of regularities:

$$F(z)= \Sigma\ \alpha_j\ R_j(z) \qquad (1)$$

where, α_j is the j-th weighting coefficient, and $R_j(z)$ is the j-th regularity.

A set of coordinates $z = (z_1, z_2,...,z_n)$ is obtained minimising F with a minimisation algorithm [27]. The resulting inflated model is supposed to fit all regularities, and, hence, to be the desired model. However, mathematical formulation of cues (regularities) led to a poorly defined objective function, which causes global minimum to be very difficult to find. Figure 3 can illustrate how different lengths in the initial step of a descent optimisation algorithm led to three different solutions. One of the design subspaces (i.e. the values adopted by the Objective Function for z coordinates of every pair of vertices), shows many local minima, very close from each other.

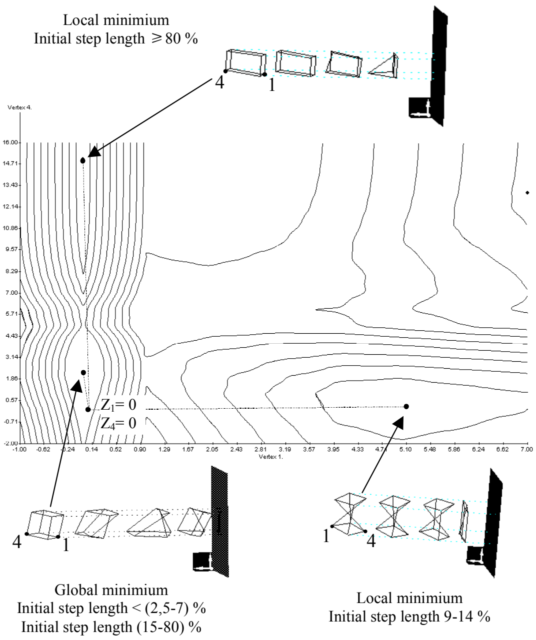


Figure 3: *Some local minimum in a design sub-space.*

Consequently, heuristic approaches to begin the optimisation from tentative models are proposed to avoid falling into local minima.

3.3. Initial solutions

Up to now, the usual strategy to select one initial solution is the simplest one of making all Z coordinates equal to 0 (i.e. the departure 2D graph is used as initial solution). Unfortunately, the departure graph is a local minimum, where many regularities are trivially satisfied. For instance, a loop composed by some edges and supposed to correspond to a plane face in the model, always determine a trivial plane face in the graph.

The strategies developed to “escape” from this trivial optimum are all based on heuristic rules to get “tentative” initial models, and all of them fall into two categories: iterative inflation and direct generation.

Iterative Inflation techniques are based on “False Regularities”, or heuristic rules supposed to be satisfied by the final model, and not related to cues in the original image. They need not necessarily be satisfied by every model; but they allow us to escape from the trivial optimum of the figure.

Marill [3] proposed the first Iterative Inflation method by defining the objective function with just one component: MSDA Principle (Minimum Standard Deviation of Angles, or maximum equality of each three-dimensional angle of a model). The MSDA is not a true regularity, because it does not reflect any properties of the image that must correspond to some properties in the searched model. It is a heuristic rule, based on the fact that regular and convex polyhedrons have a tendency to accept this assumption, consequently it seems easy to guess that approach will be well accepted just by some regular polyhedrons (equiangular 3D wire frames).

Leclerc and Fischler [4] introduced a regularity of Face Planarity, and found that, obviously, it was trivially satisfied in the 2D figure. Hence, they proposed an objective function with a linear combination, balanced by a parameter λ, in this way:

$$F = F_A + \lambda\, F_I + (1\text{-}\lambda)\, F_O \qquad (2)$$

Where F_I (the "inflation" part) includes a set of conditions that are not trivially satisfied in the initial solution (z=0), neither are true regularities. Hence, do not need to be satisfied in the optimum. F_O (the "optimisation" part) contains those *true* regularities that are trivially satisfied in the initial solution. Finally, F_A includes all those regularities that can be always applied, because they are not trivially satisfied in the initial solution.

The strategy allows the optimisation algorithm to escape from trivial solution using inflation criteria, while guarantying that the final solution depends only on true regularities. Nevertheless, undesired oscillations can happen, causing an ill working of optimisation algorithms. In addition, falling into some local minimum during inflation can result from small changes in tuning parameters, making the escape very difficult for the optimisation algorithm.

The other approach is defining a deterministic process to obtain a tentative 3D model. This is the case of the "preliminary reconstruction" proposed by Lipson and Sphitalni [5], and based on analysing the angular distribution graph of lines, to obtain the prevailing angles, and hence the main directions. When three prevailing angles are obtained, the orthogonal perceptual rule ("those angles must correspond to three orthogonal directions") is applied to obtain a tentative model. The process begins identifying the main directions. Then a model is built assigning a coordinate z=0 to an arbitrary node (what is equal to fix the height of the pattern regarding the plane of the image, for what doesn't suppose loss of generality). The process already advances assigning coordinates Z to nodes connected to those already known. If the shape has an angular distribution with exactly three main directions, the assignment of coordinates can spread to all the nodes, giving place to a good previous model.

Direct inflation approaches clearly depend on the model topology, but, in our experience, are easy to implement, and improve the optimisation process yet with simple descent algorithms.

4. Level inflation

The reconstruction by levels-inflation of vertices is a simple heuristic method of *direct* inflation (non iterative), where a set of z coordinates is determined depending on its "typologies". The vertex typologies have already been used in labelling approaches to validate the figures to reconstruct [9-14]. However, typologies are not the key of the proposed method, but it consists on accepting that there is a high degree of correspondence among the typology of each vertex in the figure and the z coordinate of its corresponding vertex in the model. Therefore, it is supposed that the typologies of the vertices can be ordered in sequential "levels", where increasing levels match to growing z coordinates. In other words, the approach is rooted on vertex labelling methods, but only a simple classification of vertices is done (and no one check about correctness of the graph is satisfied). Vertices are classified according to their typology. Different "levels" are assigned to each vertex typology. This means a relative z coordinate is supposed to fit to each vertex typology. Then heuristic z coordinates are assigned to every level, and a tentative model is so obtained.

4.1. Typologies and levels

The correspondence between typologies and levels can be intuitively justified examining the figure 4, where the departure image (contained in the XY plane) and the psychologically valid model of a cube are shown. It may be seen how the six levels have been labelled from 0 up to 5.

It must be noticed that edges that are "vertical" in the departure image (parallel to the vertical axis) are inclined in the final model. This effect is not accidental, and makes the difference between perspective line drawings and orthographic views. Perspectives are obtained by projecting the object after arbitrarily rotating it, because, by preventing their faces and edges from being parallel to the projection plane, image superposition of vertices, edges and faces are avoided.

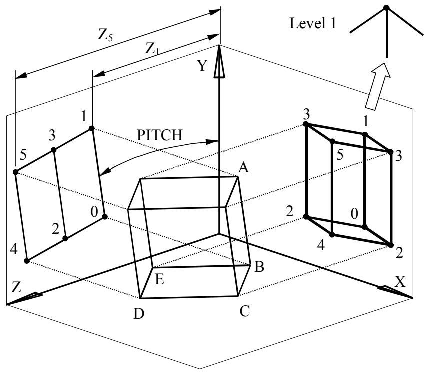
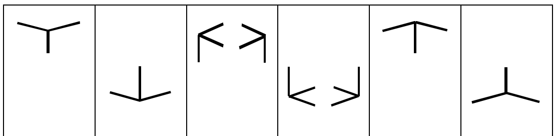


Figure 4: *Basic Model for the vertex typology definition.*

Obviously, the assignment of z coordinates to levels depends on the "pitch" of the model. Pitch angle is clearly visible in the lateral view YZ. As far as the pitch angle is free, any variation gives different projections of the same object. Bowing a little more the object of the figure 4 (increasing their pitch), z coordinates of vertices labelled 3 would become greater than z coordinate of level 4 vertices, although their typologies (their appearance in the image) would not change until a very big change occurs.

The "yaw" and "roll" parameters (to continue with the analogy of the nautical terms) modify the possible perspective views of an object as well as pitch do. The yaw angle is measured from the Z axis to the horizontal projection of line BD. We observe that when this angle clearly differs from zero, the "symmetric" vertices (for instance C and E) have different z coordinates.

Our psychological assumption was that the observer tends to make the smallest pitch and yaw movements, as a compromise to get a general point of view while keeping symmetry. In addition, roll is left zero to maintain model verticality. The resulting six defined typologies are shown in table 1 ordered in growing z values.



Level 5	Level 4	Level 3	Level 2	Level 1	Level 0
"Y"	"W"	"Lateral inverted W"	"Lateral W"	"Inverted W"	"Inverted Y"

Table 1: *Classification of levels of vertices.*

One additional consideration is that both positive and negative pitch angles give psychologically plausible solutions, known as Necker reversals. We opted for a positive pitch (i.e. the upper face is visible). This choice has a rough geometrical meaning, when the observer is in front of the model, the model is being observer from above, and the origin is in the "ground" or base plane. Furthermore, this assumption do not mean loss of generality, because changing all z coordinates from positive to negative, the reversal is obtained (figure 5).

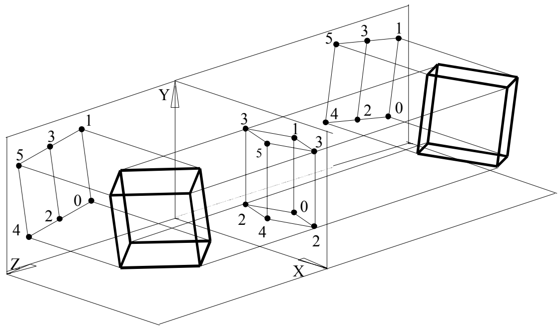


Figure 5: *Two Necker-reverse inflated modes.*

4.2. Correspondence between z coordinates and levels

Heuristic rules are applied to assign values to the z coordinates of levels.

First, we assume that the figure do maintain the proportion among the three main dimensions of the object (wide, high and deep). That is to say, it is assumed that the designer deliberately respects the proportions, by choosing the appropriate point of view to show the real object proportion or disproportion. Therefore, we can suppose that the range of depth ($\Delta z = Z_{max} - Z_{min}$) is similar to width ($\Delta x = X_{max} - X_{min}$) and height ($\Delta y = Y_{max} - Y_{min}$). Hence, as seen in figure 6, we adopt the value:

$$\Delta z = \text{maximum}\ (\Delta x, \Delta y) \qquad (3)$$

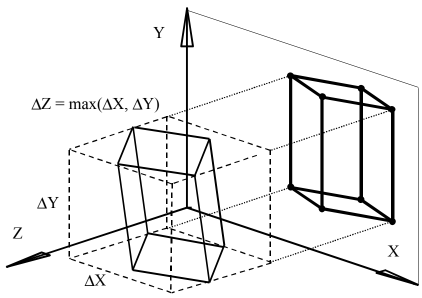


Figure 6: *Proportionality assumption.*

In fact, the "minimum box" orientation is different to the model orientation, due to the pitch, yaw and roll suffered by the object (as can be clearly seen in the example of figure 6). Hence, a proportional box is not equivalent to a proportional model. Nevertheless, we assume that a proportional box induces a rough proportional object, because a proportional box is easy to define, while the rules to determine a proportional model would depend on the particular typology every model has.

The z coordinate of one vertex can be fixed to an arbitrary value without loss of generality, since the figure is moved but not changed (it is a rigid solid translation). Therefore, z coordinate can be fixed to zero for the lower level vertices, and to maximum (Δx,Δy) for the upper level vertices.

Uniform gradation is the simplest strategy to assign z coordinates to intermediate levels:

$$z_i = i\,(\Delta z/n) \qquad (4)$$

Where n is the number of level jumps (total number of levels minus one), whose value is 5 according to table 1 classification.

The *fixed* levels-inflation so defined gives good results in convex polihedral with a high degree of regularity (figure 7). As a consequence, the subsequent optimisation process is unnecesary in some cases.

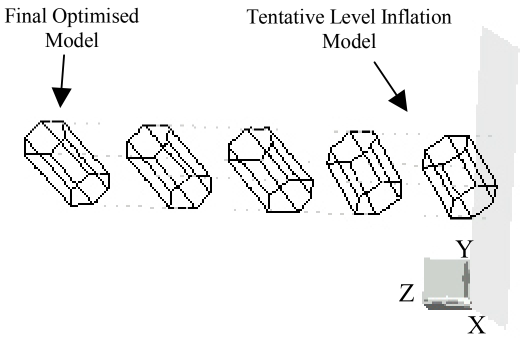


Figure 7: *Good tentative model obtained by fixed levels inflation, and subsequent optimization.*

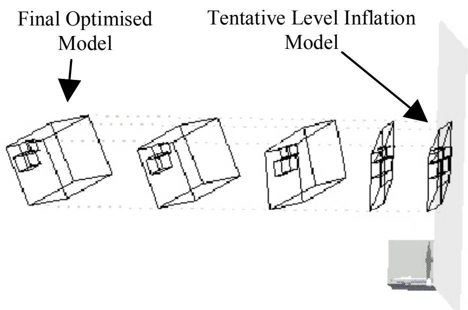


Figure 8: *Poor tentative model obtained by levels inflation, and subsequent optimization.*

Poor (clearly distored) models appear in other cases (figure 8). Nevertheless, they are proved to ve valid as tentative models for a subsequent optimisation process.

The tentative model in figure 8 results from fixed levels-inflation of figure 9. Its distortion is produced because vertices A and B have the same typology (inverted Y), in spite of having obvious different z coordinates. This is because the model is a non-convex polyhedron with a “fissure” or step.

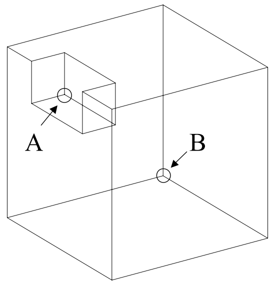


Figure 9: *Polyhedron with fissure.*

This can be solved by an *incremental* levels-inflation where z coordinates of intermediate levels are do not only depend on its absolute typology, but on the relative typology (the difference of typologies between the calculated vertex and another vertex connected to it), and the distance to the vertices to which it is connected. This proposal is again justified assuming the psychological tendency to proportion.

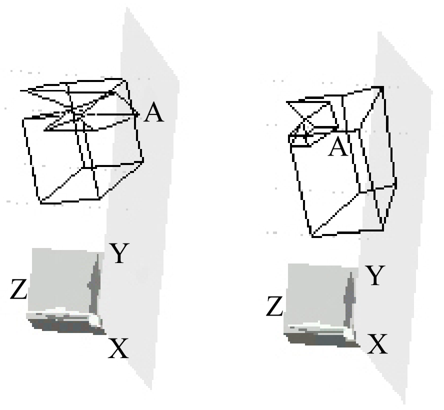
Consequently, the Z_P coordinate of a vertex P is incrementally obtained, relative to z coordinate of another vertex Q, to which P is connected by an edge:

$$Z_P = Z_Q + PQ * (Level_P - Level_Q) * \Delta z/n \qquad (5)$$

Where, PQ is the length of the projection of the edge connecting P and Q. $Level_P$-$Level_Q$ is the increment (number of jumps) from one level typology to the other. Finally, Δz was obtained in equation (3), and n is the maximum number of level jumps.

The *incremental* levels-inflation described above, can be easily extended to determine all vertices by successively calculating as much vertices as necessary. A spanning *propagation tree* [28] is used both to go from general (big) to particular (small) details, and to ensure repeatability in the tentative inflation. We use a particular version of Kruskal algorithm, departing from the most central level 0 vertex (the one closer to the figure barycentre) and following to the vertex connected with the actual through the longest edge not yet visited.

For concave models, this incremental method seems to give better results that fixed levels. For instance, the incremental inflation of vertex A in figure 10b is clearly less distorted than in the fixed inflation of figure 10a.



a) *b)*

Figure 10: *Polyhedron with fissure: a) inflated by fixed levels b) inflated by incremental levels.*

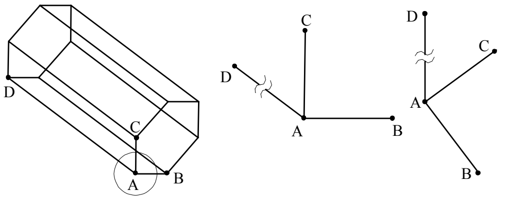
5. Extensions to the method

To generalize the approach, we introduced two strategies to cope with particular cases, and one strategy to take advantage of hidden edges.

5.1. Detection of typologies and predominant edges

The levels classification shown in table 1 was obtained supposing the predominant edge of each typology to be vertical. If we analyse the figure 11, it can be clearly observed that the vertex A in its initial orientation (figure 11b) would be of level 4. Although applying a rotation to make vertical the predominant direction, it should be classified as level 2 (figure 11c).

Therefore, to extend the approach to other cases, our algorithm detects the predominant direction and performs the vertex matching after the rotation has been neutralized.



a) b) c)

Figure 11: *Detection of dominant edge orientation and vertex alignment.*

Obviously, the concept of predominant direction depends on the object typology. Therefore, the algorithm to neutralize rotation needs the previous detection of the object typology.

Heuristic criteria to detect such typologies were adopted, as summarized in table 2.

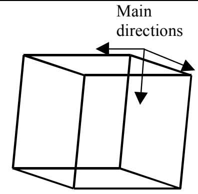
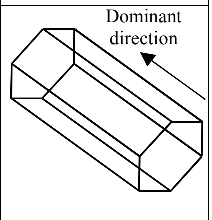
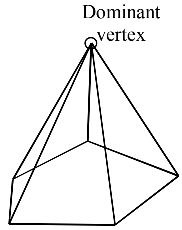
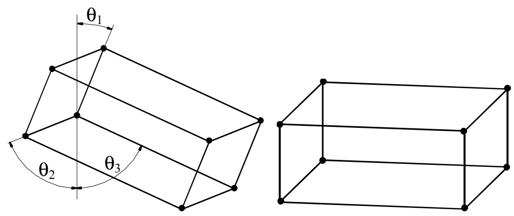
TYPOLOGIES OF POLYHEDRAL MODELS	**EXAMPLES**
Normalons One figure is considered to represent a "normalon" whenever it contains three, and only three, dominant directions. I.e. when almost all segment lines are oriented in the same three directions. To quantify the "almost all" we have tuned a numerical parameter of at least 90% of line segments oriented in one of the three dominant directions.	
Prisms One figure is considered to represent a prism whenever one single direction is dominant In other words: when the number of edges parallel to one direction is more than 10% greater than edges parallel to any other direction.	
Pyramids One figure is considered to represent a pyramid whenever one single vertex is dominant In other words: when the number of edges connected to one vertex is more than 10% greater than number of edges connected to any other vertex.	

Table 2: *Classification criteria, after image properties.*

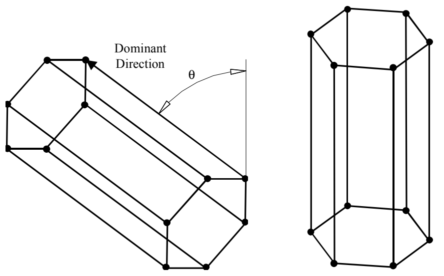
Once a *normalon* is detected, the main direction closer to 90º (preferred) or 270º acts as predominant. For instance, in the figure 12, $\theta_1 < \theta_2$ and $\theta_1 < \theta_3$, then θ_1 is chosen as the neutralizing rotation angle.



a) b)

Figure 12: *Detection of dominant edge in a normalon model: a) Original orientation b) Rotation to see the effect of neutralization.*

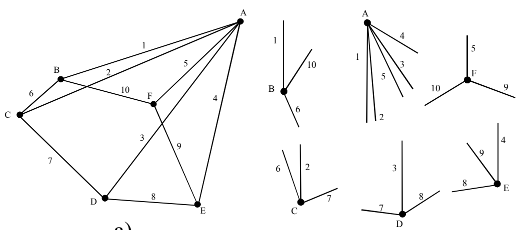
When an object of *prismatic* typology is detected, the departure image is rotated to get a vertical predominant direction (figure 13).



a) b)

Figure 13: *Dominant edge in prismatic model: a) Original orientation b) Rotation to see the effect of neutralization.*

In the case of pyramidal typology, the edge of the pyramid is considered as predominant (vertical orientation). Hence, independent neutralizations are done for every vertex of the base of the pyramid. They are rotated to align the predominant edge (the one connected to the predominant vertex) with 90º or 270º orientations (figure 14).

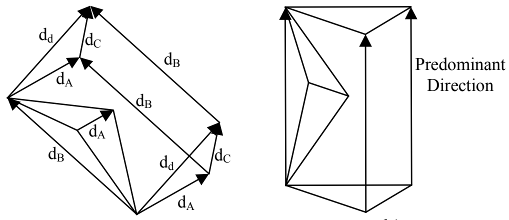


a) b)

Figure 14: *Dominant vertex in a pyramidal object a) Original orientation b) Independent rotation of vertices.*

Finally, all those that cannot be considered as objects of some of the previous typologies, are considered *indefinite*. In such cases, the most frequent direction is selected as predominant. In the case where two or more orientations appear with the same frequency, the one closer to 90º (preferred) or 270º acts as predominant.

In the figure 15a, both d_A and d_B appear three times. However, d_B is closer to the vertical, and therefore is selected as predominant edge.



a) b)

Figure 15: *Detection of dominant edge in an indefinite object: a) Original orientation b) Rotation to see the effect of neutralization.*

5.2. Vertices with more than three edges

An extension to fit in the levels typologies those vertices where more than three edges meet was also done. When four or more edges meet in a vertex, three edges are selected in a two steps process:

- The two edges encompassing the rest are selected (edges 1 and 5 of the figure 16).
- The third edge will be the one closer to the bisecting line of the angle determined by the two edges chosen in the previous step (edge 2 of the figure 16).

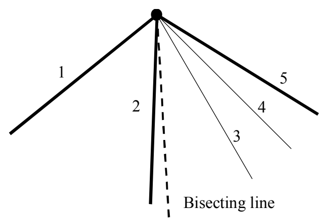


Figure 16: *Selection of edges that define the typology of a vertex.*

5.3. Hidden edges

The levels inflation can be improved when hidden lines are specified in the departure graph. The edges visibility allows enlarging the typologies of vertices defined in the table 1. Three sub-levels are considered to differentiate among hidden vertices (if all the meeting edges are hidden), partially hidden vertices (if visible and hidden edges meet), and visible vertices (if all the meting edges are visible). Obviously, hidden edges are assigned the smallest z coordinates, visible edges are assigned the greater, and partially hidden are assigned intermediate coordinate values.

The expanded typologies classification is shown in the table 4.

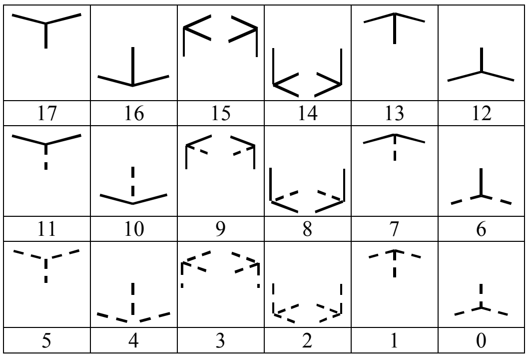


Table 4: *Extended typologies of levels, with hidden edges.*

6. Conclusions

One heuristic strategy to get inflated models was presented as an alternative to existing line-drawing reconstruction approaches. It has been implemented and tested on an application called REFER, that is being developed by the authors.

According to our experience, the heuristic levels-inflation seems to be a good approach to get departure or tentative models to improve efficiency in the regularities-based optimisation process.

In addition, models very close to the psychologically plausible one are obtained when applied to some particular kind of models. Moreover, a strategy to automatically detect such typologies was described. Hence, final good-quality models can be obtained to a reduced cost for such typologies.

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