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Technical Reports. Ref. 09/2015

Study on perceptually-based fitting elliptic arcs

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Abstract
This Technical Report revisits the problem of fitting the strokes of a sketch into elliptical arcs. Our purpose is to calculate a reasonably good and very fast fit applying a perceptual approach. Hence, the experiments carried out to determine how people perceive elliptical arcs in sketched strokes are described in detail, and the main conclusions are derived.

Index Terms: Computer-Aided Sketching, Stroke recognition, Elliptical arcs, Perceptual fit

1. INTRODUCTION

For conceptual product design, most designers still prefer the flexibility of pencil and paper rather than the constraints of Mechanical CAD packages [1]. This is unlikely to change until computer-aided shape design meets the requirements collated by van Dijk [2]: data entry should be easy and natural, both when creating and when modifying shapes; hand movements should be directly related to the shape, either by sketching in 2D or sculpting in 3D; the system should be as flexible as possible, allowing the user to sketch imprecise input, to add as much or as little numerical information as is required, and to zoom in and out to add as much or as little detail as is required; the system should allow the user to view alternative designs, including alternative interpretations of the same sketch. In addition, switching between these various modes should be transparent, implemented in such a manner that it does not disrupt the creative thought process [3].

In this context, there are two major reasons why ellipse-fitting may need to be repeated.

Firstly, the problem of *segmentation* remains unsolved: how do we divide sequences of points into strokes such that each stroke represents a single straight line or elliptical arc? It is not even clear when we should segment. Fitting lines before segmenting results in *macrolines*, in which several true lines or curves are grouped together (Figure 1 left). Segmenting strokes each time a candidate corner is detected results in *microlines*, as the segmenter wrongly interprets undulations and oscillations as true corners (Figure 1 right). We suggest that segmentation and fitting should run as parallel processes, passing information to one another until they converge to a solution. In such an approach, ellipse-fitting will be invoked each time the segmentation process identifies a candidate ellipse, so fast ellipse-fitting is a necessity.

Figure 1: Fitting a stroke into a single macroline (left), two segmented lines (centre) and many microlines (right)

Secondly, humans tend to interpret the whole scene to gain general knowledge before making a final interpretation. For example, in Figure 2 left, it is not clear what either of the highlighted strokes represents, but it is clear that they represent the same thing as the depicted object is an extrusion. Only when we take the two highlighted strokes together does it become clear that their best interpretation is an elliptical arc.

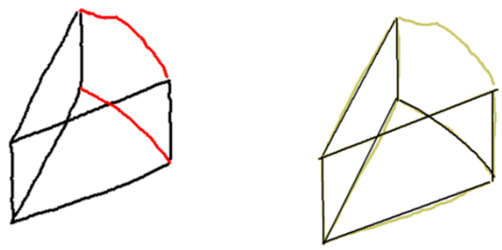

Figure 2: Context, not geometry, makes upper red stroke and arc

Our interest is to apply perceptual criteria to resolve the ellipse-fitting problem. But, to date studies in the field of visual perception rarely provide sufficient detailed information to develop an algorithmic approach to replicate human perception. On the other hand, most of the current fitting algorithms are time-consuming and add a geometrical precision which is unnecessary for interpretation of sketches. Certainly, some perceptually-oriented algorithms exist. But, from our point of view, claiming that human perception is the goal is not enough. Approaches must be designed to work in a similar way to human perception. Besides, they must also be tuned to reply as close as human perception as they can.

In other works, algorithms should accept what humans accept, should reject what humans reject, and should doubt where humans doubt.

To determine the limits of acceptance of humans, we need to ask humans, i.e. we need experiments asking individuals of representative populations. Next section describes an experimental approach to determine how humans perceive shapes embedded in sketched strokes. Graphical displays of experimental results follow Bertin's recommendations for graphical information presentation [4].

2. EXPERIMENTS

To determine the limits of human perception, we must ask humans what they perceive, and the only scientific strategy which has proved at all useful in determining what humans perceive is performing experiments with groups of humans who are then interviewed to make their perceptions explicit (e.g. [5]).

We performed four such experiments to identify which strokes humans perceive as depicting elliptical arcs, which they consider cannot be elliptical arcs, and how confident they are in their judgement. The experiments were designed to validate or reject the following hypotheses:

1. Humans categorise sketched elliptical arcs as *good*, *average* or *poor* quality, and these three categories are not further subdivided into subcategories.
2. Perception is stricter when perceiving arcs encompassing large angles (so the angle covered by an arc is an influential parameter in human perception).

The rule for ending data collection was collect valid replies from at least 20 subjects, or the maximum that could be collected in a period of time (usually a school day).

To determine whether the results are hazardous or not, and the level of significance of the test, we used binomial distribution, which is the discrete probability distribution of the number of successes in a sequence of n independent *success*/*failure* experiments. In our experiments binomial distribution is defined as follows: *success* occurs when strokes are classified by subjects in the predicted group (*good*, *average* or *poor*), and *failure* is when they are classified in a different group.

In experiment #1, subjects compared strokes against given arcs. In experiment #2 they compared strokes against their own mind's-eye arcs. The conclusion from these two experiments support our first hypothesis: humans perceive that strokes depict *good*, *average* or *poor* arcs, regardless of whether or not they are given a pattern to compare the stroke with.

Experiments #3 and #4 show that subjects are stricter with large strokes and less confident with short strokes, as shorter strokes convey less perceptual information. Hence, relaxing evaluation criteria for short strokes mimics human perception.

In experiments #1 and #2, most subjects interviewed were from technological background, with a few with only basic or no technological background. To minimise the possibility of common learnt behaviour among different subjects, those from a technological background included teachers of different courses (who studied in different universities) and students from different engineering courses (taught by different teachers). In experiments #3 and #4, all subjects were students on engineering courses.

2.1 Experiment # 1

We created a set of twelve A6 sheets strokes depicting increasingly imperfect elliptical arcs (Figure 3). The sheets also included the best-fit elliptical arc for the stroke.

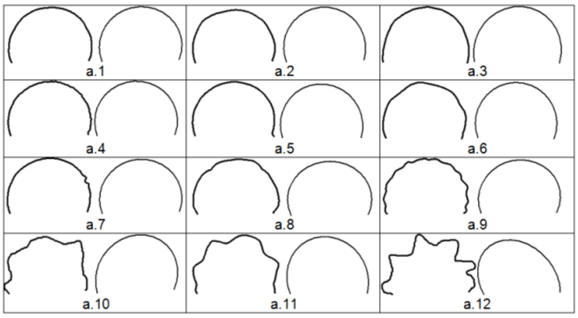

Figure 3: Set of strokes used in experiments 1 and 2 together with their suggested fits.

Each of 32 subjects was given the 12 sheets in a random order and asked to reorder them in descending order of similarity between stroke and elliptical arc.

Our results (Table 1) confirm that the subjects distinguish three groups: *good*, *average* and *poor* quality sketched elliptical arcs. 25 subjects out of the 32 classified Strokes a.1, a.2 and a.3 as *good*; Strokes a.6, a.7 and a.8 as *average*, and Strokes a.9, a.10, a.11 and a.12 as *poor*. The only unexpected result was that stroke 5 was usually perceived as *good* (90.6%) while stroke 4 was usually considered *average* (84.4%); this result is statistically significant ($p(X \geq 25) = 0.0011$, in the binomial distribution $B(32, ½)$, and assuming an alpha level of $\alpha = 0.01$). This example shows that the classification initially proposed by the authors was erroneous for Strokes 4 and 5. Hence, benchmarking algorithms according to the particular criteria of their authors is a potentially bad strategy. Instead, benchmarks obtained interviewing a representative population are mandatory to test algorithms aimed at mimicking human perception.

Table 1: Arc strokes as ordered by subjects in decreasing fitting with their best fits

Subject	Stroke a.1	a.2	a.3	a.4	a.5	a.6	a.7	a.8	a.9	ı.10	ı.11	a.12
1	3	1	5	2	7	4	8	6	9	10	11	12
2	1	3	5	2	4	7	6	8	9	11	10	12
3	1	3	2	5	8	4	6	7	11	9	10	12
4	1	5	3	2	4	7	6	8	9	10	11	12
5	3	1	5	4	7	2	6	8	9	11	10	12
6	3	2	1	5	6	4	7	8	9	11	10	12
7	3	1	2	5	6	4	8	7	9	11	10	12
8	3	2	1	5	4	7	8	9	6	11	12	10
9	3	1	2	5	6	7	4	8	9	11	12	10
10	1	3	2	5	4	7	8	6	9	11	10	12
11	5	3	2	1	4	6	7	8	9	10	11	12
12	1	3	2	5	4	6	8	7	9	11	10	12
13	1	3	2	5	4	6	8	7	9	10	11	12
14	5	1	2	3	7	4	6	8	9	11	10	12
15	1	5	3	2	4	6	8	7	9	11	10	12
16	3	1	5	2	4	6	8	7	9	11	10	12
17	3	1	2	5	4	6	8	7	9	10	11	12
18	1	5	3	2	7	4	6	8	9	10	11	12
19	1	4	5	3	2	8	6	9	7	11	10	12
20	1	3	5	2	4	7	8	6	9	11	10	12
21	1	5	3	4	7	2	6	8	9	11	10	12
22	1	3	5	2	4	7	6	8	9	11	10	12
23	3	1	4	5	2	6	9	8	7	11	12	10
24	1	3	2	5	4	6	8	7	9	10	11	12
25	1	2	5	3	4	8	6	7	9	11	10	12
26	1	5	3	2	4	6	7	8	9	11	10	12
27	1	3	4	2	5	7	6	8	9	11	10	12
28	1	2	5	3	4	7	6	8	9	11	10	12
29	1	3	2	6	8	4	5	7	9	11	10	12
30	1	3	2	6	5	4	7	8	9	11	10	12
31	5	1	3	2	7	4	6	8	9	10	11	12
32	1	5	2	3	7	4	8	6	9	11	10	12

2.2 Experiment # 2

In experiment #2, we asked another group of subjects to order the strokes of Figure 5 in descending order of perceived ellipticality *without* seeing their theoretical best fit. We collected a total of 30 questionnaires (Table 2). Even without a pattern for comparison, most subjects (20 out of 30) distinguished between *good*, *average* and *poor* quality sketched elliptical arcs, so the significance is less certain ($p(X \geq 20/30) = 0.049$).

2.3 Experiment # 3 to mimic human perception

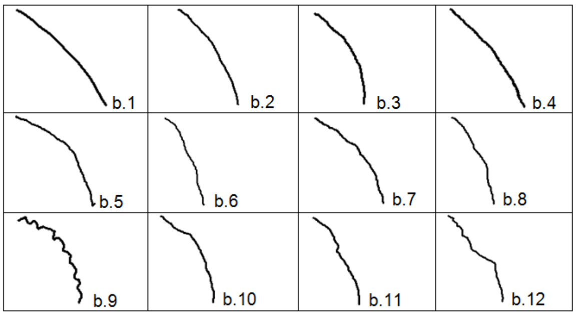

Figure 4: Set of short arcs for Experiment 3

To test the hypothesis that humans are less strict in classifying short strokes (those covering small angles of an ellipse) since they convey less perceptual information, we created a set of strokes representing short elliptical arcs with different degrees of imperfection, as shown in Figure 4.

Table 2: Arc strokes as ordered by subjects in decreasing "ellipticality" (Experiment 2)

Subjects	Stroke a.1	a.2	a.3	a.4	a.5	a.6	a.7	a.8	a.9	ı.10	ı.11	ı.12
1	1	5	3	2	4	6	8	7	9	11	10	12
2	3	1	2	5	4	6	8	7	9	10	11	12
3	3	2	1	5	4	7	6	8	9	10	11	12
4	1	5	3	2	7	4	6	8	9	11	10	12
5	1	3	5	7	4	2	6	8	9	11	10	12
6	5	1	3	2	4	6	8	7	9	10	11	12
7	5	3	1	2	7	4	8	6	9	11	10	12
8	1	3	2	6	5	4	7	8	9	11	10	12
9	1	3	2	5	7	4	6	8	9	11	10	12
10	1	3	5	4	7	2	8	6	9	11	10	12
11	1	3	5	2	7	4	8	6	9	11	12	10
12	3	5	1	2	4	7	6	8	9	10	11	12
13	1	2	3	5	4	6	7	8	9	11	10	12
14	3	5	4	2	1	8	7	6	9	11	10	12
15	5	1	2	3	4	7	6	8	9	11	10	12
16	3	1	5	4	2	6	7	8	9	11	10	12
17	1	3	2	6	4	5	7	8	9	10	11	12
18	1	3	2	5	4	6	8	7	9	11	10	12
19	3	1	5	4	2	6	7	8	9	11	10	12
20	1	3	2	4	5	6	8	9	7	11	12	10
21	1	3	5	4	2	7	6	8	11	10	9	12
22	1	2	4	3	5	6	8	7	9	11	10	12
23	2	1	3	5	4	6	8	7	9	10	11	12
24	1	3	2	4	5	6	8	7	9	11	10	12
25	3	1	5	2	4	6	7	8	9	11	10	12
26	1	3	2	4	5	7	8	6	9	11	10	12
27	1	4	3	5	2	9	8	6	7	11	12	10
28	1	3	2	5	7	4	6	8	9	10	11	12
29	3	2	1	4	6	5	8	7	9	11	12	10
30	3	1	2	4	6	5	8	7	9	11	10	12

Table 3: Short arc strokes as ordered by subjects in Experiment 3

Subjects	Positions b.1	b.2	b.3	b.4	b.5	b.6	b.7	b.8	b.9	b.10	b.11	b.12
1	1	2	4	3	7	6	8	11	10	5	9	12
2	1	3	5	7	2	4	11	8	10	9	6	12
3	3	5	2	1	4	7	6	11	10	8	12	9
4	2	1	4	3	7	6	5	8	11	10	12	9
5	3	5	1	4	2	7	10	6	8	11	12	9
6	1	3	2	4	7	5	6	11	8	10	12	9
7	5	3	2	4	1	7	11	6	8	10	12	9
8	2	3	4	7	8	6	10	11	5	12	9	1
9	3	5	2	7	1	4	6	10	11	8	12	9
10	1	4	2	5	3	7	6	8	11	12	10	9
11	1	2	3	4	7	5	6	8	11	10	9	12
12	3	2	7	5	1	4	6	8	11	9	10	12
13	3	4	1	2	5	7	6	10	8	11	9	12
14	3	9	7	5	1	10	11	2	8	4	12	6
15	3	2	1	4	5	6	7	8	11	10	9	12
16	2	1	3	7	4	5	6	8	9	11	10	12
17	5	7	3	9	2	1	10	6	11	8	12	4
18	3	5	1	2	10	7	11	8	4	6	9	12
19	1	2	4	3	5	6	7	8	11	10	12	9
20	3	2	7	1	5	4	8	6	9	11	10	12
21	3	5	9	7	2	1	4	6	10	11	8	12
22	1	2	4	3	5	7	11	8	6	10	12	9
23	1	2	3	4	5	7	6	11	10	8	12	9
24	1	3	2	4	5	7	10	8	6	11	12	9
25	1	2	4	5	3	7	6	11	8	10	12	9
26	1	2	3	7	4	5	6	11	8	10	12	9
27	3	5	1	7	10	2	8	6	4	12	11	9
28	3	5	7	2	1	4	10	9	11	8	6	12

We presented the strokes on four different randomly-ordered A4 questionnaires. We distributed 28 questionnaires and collected 28 responses. Table 3 shows the results.

As we hypothesised, the distinctions between *good*, *average* and *poor* quality strokes are not as clear as with longer strokes. Subjects still perceive very *good* and very *poor* strokes, but the *average* group becomes fuzzy: 18 out of 28 subjects perceived Strokes b.1, b.2 and b.3 as *good* ($p(X \geq 18/28) = 0.092$) and 24 considered b.9 and b.12 *poor* ($p(X \geq 24/28) < 0.001$).

We conclude that subjects are less certain in classifying short strokes than they are with long strokes. Subjects seem to be less strict with small irregularities or oscillations, checking only whether the underlying stroke has enough radius of curvature to be perceived as an elliptical arc. But they still penalise undulations, and strongly penalise apparent corners caused by high frequency oscillations of small amplitude as in b.9.

2.4 Experiment # 4

In experiment #4, strokes represent arcs with different lengths (from almost 360° to less than a quarter of an ellipse). For each length, we created three different quality arcs (Figure 5).

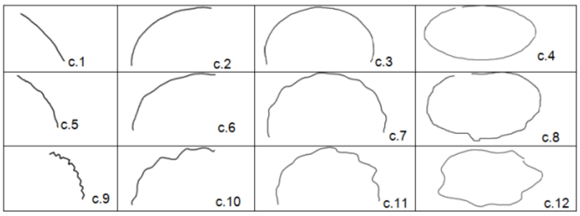

Figure 5: Set of different arc lengths for Experiment 4

As in Experiment #3, we presented the strokes on four A4 questionnaires with the strokes in different random orders, instructing the subjects to number each example in descending order of ellipticality. We presented 32 questionnaires; all 32 subjects responded. Table 4 shows the results.

The results show that subjects seems to follow a classification pattern: Strokes c.1, c.2, and c.3 were classified as *good* for 22 out of 32 subjects ($p(X \geq 22/32 = 0.025)$; Strokes c.5, c.6, and c.7 were classified *average* by 21 of polled subjects, and Strokes c.9, c.11 and c.12 as *poor* also by 21 of polled subjects ($p(X \geq 21/32) = 0.055$). However, Strokes c.4 and c.8, which represent nearly complete arcs, are the examples with the lowest frequency of perception as *good* or *average* respectively, with a notable presence at the next lower group. On the contrary, Stroke c.10 is the stroke of poor group with more presence in the average group; maybe the irregular undulations and the short length of the stroke made people were less strict. Stroke c.12, which also represents an almost complete arc with large irregularities, is the example most often perceived as *poor*.

Table 4: Arc strokes as ordered by subjects in Experiment 4

	Positions											
Subjects	**c.1**	**c.2**	**c.3**	**c.4**	**c.5**	**c.6**	**c.7**	**c.8**	**c.9**	**c.10**	**c.11**	**c.12**
1	4	3	2	6	5	8	7	1	10	11	12	9
2	1	3	2	6	5	7	4	8	10	11	9	12
3	1	3	6	2	4	5	7	8	10	11	9	12
4	4	3	2	6	5	8	7	1	10	11	12	9
5	2	3	6	1	5	10	9	7	11	4	8	12
6	2	6	3	7	11	5	10	9	4	8	12	1
7	1	2	3	5	4	6	8	10	7	11	9	12
8	4	3	2	6	1	5	7	8	11	12	10	9
9	1	5	2	3	4	12	6	9	7	10	11	8
10	2	1	3	4	6	5	7	8	10	9	11	12
11	1	2	5	6	3	10	7	9	11	4	8	12
12	3	2	4	1	6	5	7	8	11	12	10	9
13	2	5	3	1	6	4	7	8	11	12	10	9
14	1	2	4	3	5	7	8	6	11	9	10	12
15	1	4	2	3	5	6	7	8	12	10	11	9
16	1	2	5	6	3	4	10	9	7	8	11	12
17	2	1	5	3	6	4	10	7	8	9	11	12
18	3	6	2	1	5	7	11	10	9	4	8	12
19	4	3	2	1	5	6	7	11	10	12	8	9
20	1	3	4	2	12	5	7	6	11	10	8	9
21	2	3	4	1	5	6	7	8	11	10	12	9
22	3	4	2	1	5	6	7	8	10	11	9	12
23	4	3	2	1	5	6	7	9	8	10	11	12
24	1	3	4	6	2	5	7	11	8	12	10	9
25	3	2	4	7	6	1	5	8	12	11	10	9
26	3	2	1	4	5	6	7	8	11	10	12	9
27	1	3	5	2	4	6	8	7	10	11	12	9
28	1	3	2	5	6	4	8	7	11	12	10	9
29	1	3	2	4	6	5	12	11	8	7	10	9
30	1	3	8	2	4	5	6	7	12	11	10	9
31	1	2	6	5	3	12	4	7	8	11	10	9
32	4	3	2	7	11	8	6	1	5	10	12	9

Since subjects ordered the strokes not by their arc lengths but by their irregularities, we conclude that, while humans are stricter when perceiving long strokes, arc length has only a secondary influence on perception.

3. CONCLUSIONS

We have designed and implemented questionnaires to identify which strokes humans perceive as depicting elliptical arcs, and which they consider cannot be elliptical arcs.

Conclusions from Experiment #1 and Experiment #2 support our first hypothesis: humans perceive that strokes depict good, average or poor arcs, regardless of whether or not they are given a pattern to compare the stroke with.

Experiments #3 and #4 confirm our second hypothesis: humans are stricter with large strokes and less confident with short strokes, as shorter strokes convey less perceptual information.

4. ACKNOWLEDGEMENTS

This work was partially funded by financial support from the Ramon y Cajal Scholarship Programme and by the "Pla de Promoció de la Investigació de la Universitat Jaume I", project P1 1B2010-01. We wish to thank Salvador Mondragón, who collected many questionnaires from his students, and Margarita Vergara, for her contribution to statistical data treatment.

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